

Section 5.5 – Multiple-Angle Formulas

Don't worry, you do not have to memorize the following formulas, but you have to know how to use them....

Double-Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

Ex. 1) Use a double-angle formula to rewrite the expression.

$$6\cos^2 x - 3 = 3(2\cos^2 x - 1) = 3\cos 2x$$

Ex. 2) Use the figure to find the exact value of the following:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

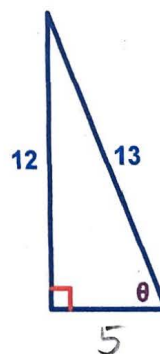
$$= 2\left(\frac{12}{13}\right)\left(\frac{5}{13}\right) = \frac{120}{169}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{5}{13}\right)^2 - \left(\frac{12}{13}\right)^2 = \frac{25}{169} - \frac{144}{169} = -\frac{119}{169}$$

$$\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} = \frac{2\left(\frac{12}{5}\right)}{1 - \left(\frac{12}{5}\right)^2} = \frac{\frac{24}{5}}{\frac{25 - 144}{25}}$$

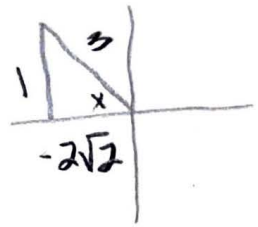
$$= \frac{24}{5} \cdot \frac{25}{-119} = -\frac{120}{119}$$



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Ex. 3) Find the exact value of the following if $\csc x = 3$, and $\frac{\pi}{2} < x < \pi$

$$\begin{aligned} \tan 2x &= \frac{2 \tan x}{1 - \tan^2 x} \\ &= \frac{2 \left(-\frac{1}{2\sqrt{2}} \right)}{1 - \left(-\frac{1}{2\sqrt{2}} \right)^2} = \frac{-\frac{1}{\sqrt{2}}}{1 - \frac{1}{8}} \end{aligned}$$



$$= \frac{-\frac{1}{\sqrt{2}}}{\frac{8-1}{8}} = -\frac{1}{\sqrt{2}} \cdot \frac{8}{7} = -\frac{8}{\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}} \right) = -\frac{8\sqrt{2}}{14} = \boxed{-\frac{4\sqrt{2}}{7}}$$

Ex. 4) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\sin 2x + \cos x = 0$$

$$2 \sin x \cos x + \cos x = 0$$

$$\cos x (2 \sin x + 1) = 0$$

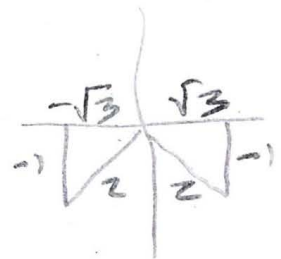
$$\cos x = 0$$

$$2 \sin x + 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = -\frac{1}{2}$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$



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Ex. 5) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\cos 2x + \sin x = 0$$

$$1 - 2\sin^2 x + \sin x = 0$$

$$-2\sin^2 x + \sin x + 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2}$$

$$\sin x = 1$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$x = \frac{\pi}{2}$$

Ex. 6) Find the exact solution of the equation in the interval $[0, 2\pi)$

$$\sin 2x \cos x = \sin x$$

$$(2\sin x \cos x) \cos x - \sin x = 0$$

$$2\sin x \cos^2 x - \sin x = 0$$

$$2\sin x (1 - \sin^2 x) - \sin x = 0$$

$$2\sin x - 2\sin^3 x - \sin x = 0$$

$$\sin x - 2\sin^3 x = 0$$

$$\sin x (1 - 2\sin^2 x) = 0$$

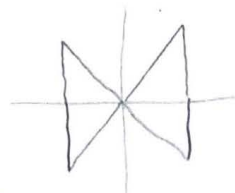
$$\sin x = 0$$

$$\sin^2 x = \frac{1}{2}$$

$$\sin x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}}$$

$$x = 0, \pi$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$



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****The signs of $\sin \frac{\theta}{2}$ and $\cos \frac{\theta}{2}$ depend on the quadrant in which $\frac{\theta}{2}$ lies****

Half-Angle Formulas

$$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$$

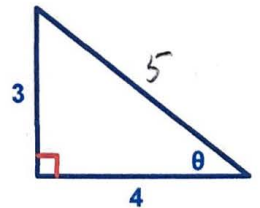
$$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$

Ex. 1) Use the half-angle formulas to simplify the expression.

$$\frac{1 - \cos 8x}{\sin 8x} = \tan \frac{8x}{2} = \tan 4x$$

Ex. 2) Use the figure to find the exact value of the following:

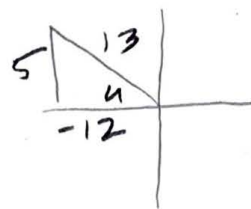


$$\begin{aligned} \sin \frac{\theta}{2} &= \pm \sqrt{\frac{1 - \cos \theta}{2}} \\ &= \pm \sqrt{\frac{1 - \frac{4}{5}}{2}} \\ &= \pm \sqrt{\frac{\frac{1}{5}}{2}} = \pm \frac{\sqrt{10}}{10} \end{aligned}$$

$$\begin{aligned} \tan \frac{\theta}{2} &= \frac{1 - \cos \theta}{\sin \theta} \\ &= \frac{1 - \frac{4}{5}}{\frac{3}{5}} = \frac{\frac{1}{5}}{\frac{3}{5}} \\ &= \frac{1}{3} \end{aligned}$$

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Ex. 3) Find the exact value of $\tan \frac{u}{2}$ if $\sin u = \frac{5}{13}$ and $\frac{\pi}{2} < u < \pi$.

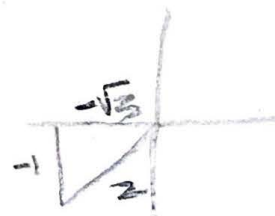


$$= \frac{1 - \cos u}{\sin u} = \frac{1 - \frac{-12}{13}}{\frac{5}{13}} = \frac{\frac{25}{13}}{\frac{5}{13}} = 5$$

Ex. 4) Use the half-angle formulas to determine the exact values of:

$$\begin{aligned} \text{a) } \sin(22.5^\circ) &= \sin\left(\frac{45^\circ}{2}\right) = \pm \sqrt{\frac{1 - \cos 45^\circ}{2}} = \pm \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} \\ &= \pm \sqrt{\frac{2 - \sqrt{2}}{2}} = \pm \sqrt{\frac{2 - \sqrt{2}}{4}} \end{aligned}$$

$$\begin{aligned} \text{b) } \cos 105^\circ &= \cos \frac{210^\circ}{2} = \pm \sqrt{\frac{1 + \cos 210^\circ}{2}} \\ &= \pm \sqrt{\frac{1 + \frac{-\sqrt{3}}{2}}{2}} = \pm \sqrt{\frac{2 - \sqrt{3}}{2}} \\ &= \pm \sqrt{\frac{2 - \sqrt{3}}{4}} \end{aligned}$$



HW: 5.5 p.415: 3, 5, 9, 13, 19, 21, 27, 35, 37, 41, 53, 55